AD No. 31612

ASTIA FILE COPY

APRIL 57-13

REPORT

DEPARTMENT OF ENGINEERING

FC

TECHNICAL STUDIES IN CARGO HANDLING - II

COMPUTATION OF DELAYS IN THE MULTI-STAGE SHUTTLE PROCESS

Richard Bellman
Yoichiro Fukuda
Maurice Pollack

UNIVERSITY OF CALIFORNIA, LOS ANGELE

REPORT NO. 57-13

TECHNICAL STUDIES IN CARGO HANDLING - II

Computation Of Delays In The Multi-Stage Shuttle Process

by

Richard Bellman
Yoichiro Fukuda
Maurice Pollack

FOREWORD

The series of reports which is entitled "Technical Studies in Cargo Handling" is primarily a working paper reporting on the progress of research or the completion of a portion of a larger investigation. This study is being published in a tentative form in order to disseminate the information as quickly as possible among the several groups who are currently working on related problems. This paper may be expanded, modified, withdrawn, or published as a report in the series entitled "An Engineering Analysis of Cargo Handling" or some other form at a later date.

The work described in this report was carried out under the supervision and technical responsibility of Russell R. O'Neill, and is part of the program in Cargo Handling. The research conducted under the sponsorship of the Office of Naval Research, Department of the Navy, was performed in the Department of Engineering, University of California, Los Angeles. L. M. K. Boelter is the Chairman of the Department.

Submitted in partial fulfillment
of Contract No. Nonr 233(07).

ABSTRACT

This report describes a Monte-Carlo approach to the calculation of delays in the multi-stage shuttle process by means of SWAC, a high-speed digital computer. Several codes were developed for SWAC to generate the random time elements, and to calculate the delays in the 2nd stage for 3- , 4- , 5- , and 6-stage shuttle processes. It was found that the 2nd stage delays did not seem to be influenced by the item number but were affected slightly by the number of stages, the delays tending to increase with increasing number of stages.

TABLE OF CONTENTS

NOTATION

$d_i(k)$	The delay to T_i incurred waiting for T_{i-1} at P_i to receive the k^{th} item (unit of commodity)
$\delta_i(k)$	The delay to T_i incurred waiting for T_{i+1} at P_{i+1} when delivering the k^{th} item (unit of commodity)
$f(t)$	Probability density function of t
f_{2j}	Frequency of discrete delay value 2j computed by SWAC
F_{2m+1}	The cumulative frequency distribution of f_{2j}
i	Subscript denoting the i^{th} stage (link)
k	The k^{th} item (unit of commodity) transported
N	The number of stages (links) in a shuttle process
P_i	The i^{th} node - the juncture of the $i-1^{th}$ and i^{th} stages (links)
$t_i(k)$	The time required for T_i to convey the k^{th} item (unit of commodity) from P_i to P_{i+1}
$t'_i(k)$	The time for T_i to return from P_{i+1} to P_i after having delivered the k^{th} item (unit of commodity) to T_{i+1}
T_i	The shuttle (transporting agent) in the i^{th} stage (link)

The above notation is consistent with the notation in Technical Studies in Cargo Handling - I. For convenience the corresponding terms which were introduced in the series, An Engineering Analysis of Cargo Handling, are included in parentheses.

INTRODUCTION

The basic recurrence relations for the delays found in the general N-stage shuttle process have been formulated by R. Bellman in [1] *. This formulation considers only one shuttle operating in each stage with no storage at the intermediate nodes. No storage refers to the requirement that the items be transferred directly from one shuttle to the next shuttle. Delays to the shuttles will occur then if the two shuttles do not arrive at the node positions simultaneously.

A consideration of the delays is important to an understanding of the effectiveness of a shuttle process, as is shown by R. R. O'Neill in [2] . An effectiveness of 1.00 is attained in a shuttle process if the delays are always zero. If the shuttle process is a stochastic process, (*i. e.*, if only the frequency distribution of the element times are known) then the delays may or may not be zero and in general will also have a frequency distribution.

The distribution of the delays might be expected to be dependent on the number of stages in the process and also on the item number. The delays to the shuttle involved in transporting the $k^{\underline{th}}$ item would be different from the delays to the shuttle involved in transporting the $k+1^{\underline{th}}$ item.

---

* Numbers in square brackets indicate references in the Bibliography.

I - RECURRENCE EQUATIONS

The basic recurrence relations for the delays found in the general N-stage shuttle process have been formulated by R. Bellman in [1] . This process is shown in Figure 1.

FIGURE 1

There are N+1 positions or nodes, P_1, P_2, ... P_{N+1}; and N shuttles, T_1, T_2, ... T_N. Each shuttle T_i operates between the nodes P_i and P_{i+1}. The last shuttle, T_N, deposits the items at the terminal P_{N+1}.

Since there is no storage provision at the intermediate nodes, the items can only be transferred directly from one shuttle to the next. A shuttle may therefore experience two types of delays which are defined as follows:

$d_i(k)$ = the delay to T_i incurred waiting for T_{i-1} at P_i to receive the $k^{\underline{th}}$ item.

$\delta_i(k)$ = the delay to T_i incurred waiting for T_{i+1} at P_{i+1} when delivering the $k^{\underline{th}}$ item.

The process time elements are defined as

$t_i(k)$ = the time required for T_i to convey the $k^{\underline{th}}$ item from P_i to P_{i+1}.

$t'_i(k)$ = the time required for T_i to return from P_{i+1} to P_i after having delivered the $k^{\underline{th}}$ item to T_{i+1}.

As given in [1], the general recurrence relations for the delays are

(1) $$d_i(k+1) = \max\left[t_{i-1}(k+1) + t'_{i-1}(k) - t_i(k) - t'_i(k) - \delta_i(k) + d_{i-1}(k+1),\ 0\right]$$

(2) $$\delta_i(k+1) = \max\left[t_{i+1}(k) + t'_{i+1}(k) - t_i(k+1) - t'_i(k) - d_i(k+1) + \delta_{i+1}(k),\ 0\right]$$

Since initially there is a stockpile of items at P_1, T_1 will experience no delay in receiving items.

(3) $$d_1(k) = 0, \quad k = 1, 2, \ldots . n$$

Also, T_N will experience no delay in depositing the items at the terminal.

(4) $$\delta_N(k) = 0, \quad k = 1, 2, \ldots . n$$

And, as the first item is transported through the process, every shuttle will experience no delay in delivering this item since initially all the shuttles are in position ready to receive this first item.

(5) $$\delta_i(1) = 0, \quad i = 1, 2, \ldots . N$$

The delay to any shuttle awaiting the first item is then simply the summation of the previous transport times.

(6) $$d_i(1) = \sum_{j=1}^{i-1} t_j(1), \quad i \geq 2$$

The delays may then be calculated for any i or k by use of (1) and (2) if the transport and return times are known.

As shown in [2], the over-all effectiveness of a shuttle process is the same as the effectiveness of any stage of this process if there is no storage provision at the intermediate nodes. This is true since the average flow of items over a period of time must be the same through every stage. The distribution of delays is important to an understanding of the effectiveness of a shuttle process (or a single stage of this process), since as given in [2]

effectiveness is defined as the ratio of the theoretical average no-delay round trip time of a shuttle to the actual average round trip time of a shuttle. This report describes the behavior of the delays, $d_2(k)$ and $\delta_2(k)$, in the 2nd stage for 3- , 4- , 5- and 6-stage processes. In each of these processes, the behavior was investigated of the delays to the 2nd stage shuttle transporting items k = 2, 3, 4, 101, 102, 103 and 104.

The recurrence relations necessary to determine these delays for any k may be obtained from (1) and (2) for N = 3, 4, 5 and 6. The particular relations used in the computation of delays are listed below for each process.

<u>3-Stage Shuttle Process</u>

(7) $$d_2(k+1) = \max\left[t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k) - \delta_2(k),\ 0\right]$$

(8) $$\delta_2(k+1) = \max\left[t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k) - d_2(k+1),\ 0\right]$$

<u>4-Stage Shuttle Process</u>

(9) $$d_2(k+1) = \max\left[t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k) - \delta_2(k),\ 0\right]$$

(10) $$\delta_2(k+1) = \max\left[t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k) - d_2(k+1) + \delta_3(k),\ 0\right]$$

(11) $$d_3(k+1) = \max\left[t_2(k+1) + t_2'(k) - t_3(k) - t_3'(k) - \delta_3(k) + d_2(k+1),\ 0\right]$$

(12) $$\delta_3(k+1) = \max\left[t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k) - d_3(k+1),\ 0\right]$$

<u>5-Stage Shuttle Process</u>

(13) $$d_2(k+1) = \max\left[t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k) - \delta_2(k),\ 0\right]$$

(14) $$\delta_2(k+1) = \max\left[t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k) - d_2(k+1) + \delta_3(k),\ 0\right]$$

(15) $$d_3(k+1) = \max\left[t_2(k+1) + t_2'(k) - t_3(k) - t_3'(k) - \delta_3(k) + d_2(k+1),\ 0\right]$$

(16) $$\delta_3(k+1) = \max\left[t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k) - d_3(k+1) + \delta_4(k),\ 0\right]$$

(17) $$d_4(k+1) = \max\left[t_3(k+1) + t_3'(k) - t_4(k) - t_4'(k) - \delta_4(k) + d_3(k+1),\ 0\right]$$

(18) $\delta_4(k+1) = \max \left[t_5(k) + t_5'(k) - t_4(k+1) - t_4'(k) - d_4(k+1),\ 0 \right]$

6-Stage Shuttle Process

(19) $d_2(k+1) = \max \left[t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k) - \delta_2(k),\ 0 \right]$

(20) $\delta_2(k+1) = \max \left[t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k) - d_2(k+1) + \delta_3(k),\ 0 \right]$

(21) $d_3(k+1) = \max \left[t_2(k+1) + t_2'(k) - t_3(k) - t_3'(k) - \delta_3(k) + d_2(k+1),\ 0 \right]$

(22) $\delta_3(k+1) = \max \left[t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k) - d_3(k+1) + \delta_4(k),\ 0 \right]$

(23) $d_4(k+1) = \max \left[t_3(k+1) + t_3'(k) - t_4(k) - t_4'(k) - \delta_4(k) + d_3(k+1),\ 0 \right]$

(24) $\delta_4(k+1) = \max \left[t_5(k) + t_5'(k) - t_4(k+1) - t_4'(k) - d_4(k+1) + \delta_5(k),\ 0 \right]$

(25) $d_5(k+1) = \max \left[t_4(k+1) + t_4'(k) - t_5(k) - t_5'(k) - \delta_5(k) + d_4(k+1),\ 0 \right]$

(26) $\delta_5(k+1) = \max \left[t_6(k) + t_6'(k) - t_5(k+1) - t_5'(k) - d_5(k+1),\ 0 \right]$

II - DESCRIPTION OF COMPUTATIONS AND RESULTS

Hypothesis and Assumptions

The effectiveness of the 2nd stage shuttle depends only on the total delay experienced by this shuttle, namely, $d_2(k) + \delta_2(k)$. Reference to relations (7) through (26) shows that each delay $d_2(k+1)$ lies on a higher level of complexity than $\delta_2(k)$, and each $\delta_2(k+1)$ higher than $d_2(k+1)$. Therefore the computations are performed in such a manner that the results may suggest the distribution of each delay separately. Specifically, 200 values of each delay, $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$, are computed for items $k = 2, 3, 4, 101, 102, 103$ and 104 in the 3-, 4-, 5- and 6-stage shuttle process.

All the delays are computed using element times, $t_1(k)$ and $t_1'(k)$, which are assumed to be obtained from the following continuous frequency function.

(27) $f(t) = 1/4e^{-1/4t}, \quad t \geq 0$

All the values of $t_1(k)$ and $t_1'(k)$ are assumed to be random variables, with functional form (27), independent of each other for all values of i and k.

Adaption of SWAC

SWAC, the high speed digital computor at the Numerical Analysis Institute, UCLA, is used to compute the desired delays. The operation of SWAC is briefly described below for the 4-stage shuttle process. The operation of SWAC for the other shuttle processes is entirely similar.

1) To compute $d_2(2)$, $\delta_2(2)$, and $d_2(2) + \delta_2(2)$ according to relations (9) through (12), SWAC generates 10 random time elements $t_1(2)$, $t_1'(1)$, $t_2(1)$,

$t_2'(1)$, $t_3(1)$, $t_3'(1)$, $t_4(1)$, $t_4'(1)$, $t_3(2)$ and $t_2(2)$. The process of generating random variables by SWAC is described in [2] on pages 27 to 32.

SWAC computes $\alpha_1 = t_1(2) + t_1'(1) - t_2(1) - t_2'(1)$, subtracts $\delta_2(1)$ which is zero in this case, from α_1, and calculates $d_2(2) = \max\left[\alpha_1 - \delta_2(1), 0\right]$. In the same way, $\beta_1 = t_3(1) + t_3'(1) - t_2(2) - t_2'(1)$ and $\gamma_1 = t_4(1) + t_4'(1) - t_3(2) - t_3'(1)$ are computed, and $\delta_2(2) = \max\left[\beta_1 - d_2(2) + \delta_3(1), 0\right]$, $d_3(2) = \max\left[d_2(2) - \beta_1 - \delta_3(1), 0\right]$, and $\delta_3(2) = \max\left[\gamma_1 - d_3(2), 0\right]$ are obtained from $\delta_3(1) = 0$.

2) To compute $d_2(3)$, $\delta_2(3)$, and $d_2(3) + \delta_2(3)$, another set of 10 random time elements $t_1(3)$, $t_1'(2)$, $t_2(2)$, $t_2'(2)$, $t_3(2)$, $t_3'(2)$, $t_2(3)$, $t_4(2)$, $t_4'(2)$ and $t_3(3)$, is required. Since $t_2(2)$ and $t_3(2)$ have already been generated in the previous set, these two elements in the new set must be replaced by the previous values in the old set. $d_2(3)$ and $\delta_2(3)$ are then computed in the same manner, using the previously obtained values of $\delta_2(2)$ and $\delta_3(2)$.

3) SWAC repeats the above procedures for $k = 4$, and punches out these results on an IBM card. To obtain 200 groups of these results, SWAC repeats the entire process 200 times.

4) To compute $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$ for $k = 101, 102, 103, 104$, SWAC starts with $k = 2$ as before, continues calculation up to $k = 104$, at which it punches out the answers only for $k = 101, 102, 103, 104$ and resets itself for another sequence of computations starting from $k = 2$. The 200 groups of delay times are obtained by 200 repetitions of these processes.

<u>Input and Output Data</u>

The exponential frequency function (27) is approximated for computational

purposes by the histogram shown in Figure 2. This histogram involves nine groups of times assuming time is a discrete variable, the mid-points of each interval all being odd integers. The area of the histogram is divided into 80 equal units of area so that the time, t, associated with each unit of area can be stored in the 80 positions in the high speed memory of SWAC.

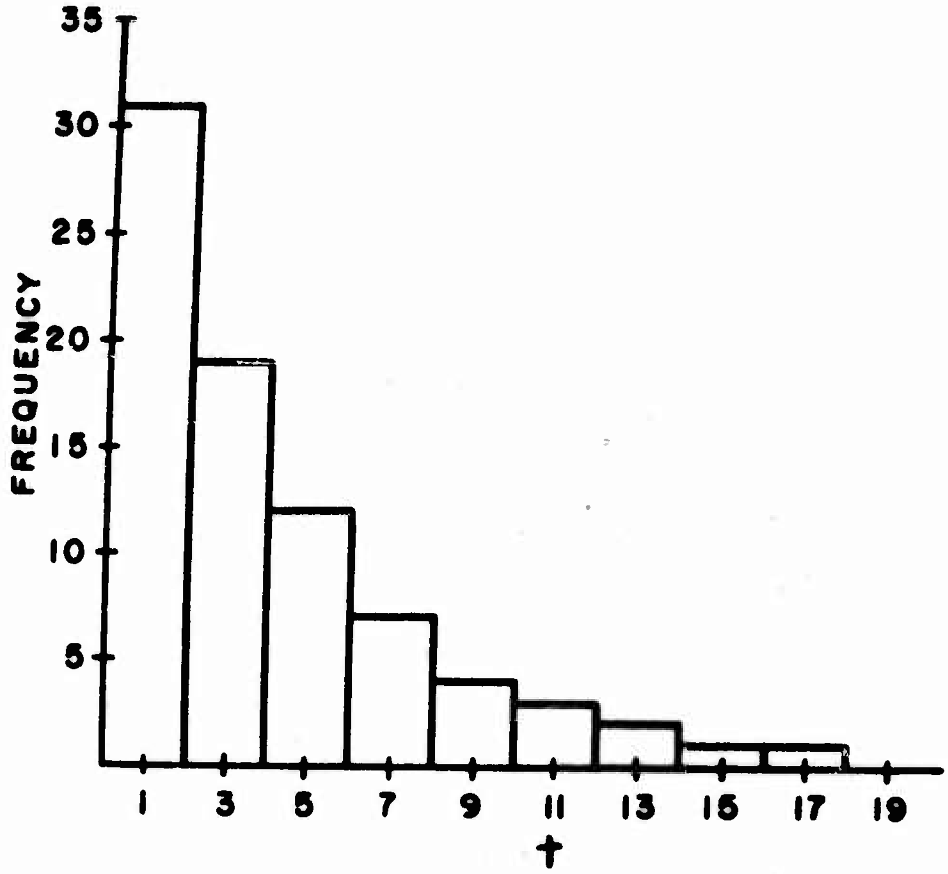
The 200 values of each delay are interpreted as being 200 random samples of the delay. The computed delays will only assume even integers since by relations (7) through (26) four values of time are always combined. The cumulative frequency distribution is defined for every type of delay as

$$(28) \qquad F_{2m+1} = \frac{1}{200} \sum_{j=0}^{m} f_{2j}$$

where f_{2j} is the frequency of the delay value 2j.
The frequencies, f_{2j}, are obtained from the 200 computed values of delay. The cumulative frequency, F_{2m+1}, is then interpreted as the frequency or probability of the delay being less than or equal to 2m+1 . These discrete distributions jump in value only at odd numbered values of delay and are therefore approximations to the actual continuous cumulative distributions.

Time Required for Coding and Computation

The coding for the 3 stage shuttle process required about 16 hours. A coding process includes planning of subroutines, drawing of a simplified flow diagram, writing out the code proper cell by cell, punching out the IBM cards, tabulating and checking. The coding for the 4- , 5- and 6-stage shuttle processes required about 10 hours for each process. Before starting the actual calculation,



STATISTIC	HISTOGRAM	f(t)
MEAN, m	4.05	4.0
STANDARD DEVIATION, σ	3.66	4.0

FIGURE 2 HISTOGRAM FOR f(t)

SWAC is used to check the code against any over-looked errors and misplanned subroutines. This type of checking required about 1 to 1-1/2 hours for each process. The actual computation to obtain the 200 groups of data for $k = 2, 3$ and 4 took about 5 minutes for each process. For $k = 101, 102, 103$ and 104, the same operations required about 10 to 15 minutes for each process.

Results

The cumulative frequency distributions as calculated by (28) are shown in Figures 3 to 14. The distributions are given for each type of delay, $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$, for the 3- , 4- , 5- and 6-stage shuttle processes. In each figure only a few representative values of k are shown since, as can be seen from the graphs, the results are apparently random in nature for different values of k.

The delays $\delta_2(k)$ tend to increase with increasing number of stages. The delays $d_2(k)$ tend to remain unchanged with change in number of stages. And, the total delays $d_2(k) + \delta_2(k)$ also tend to increase with increasing number of stages.

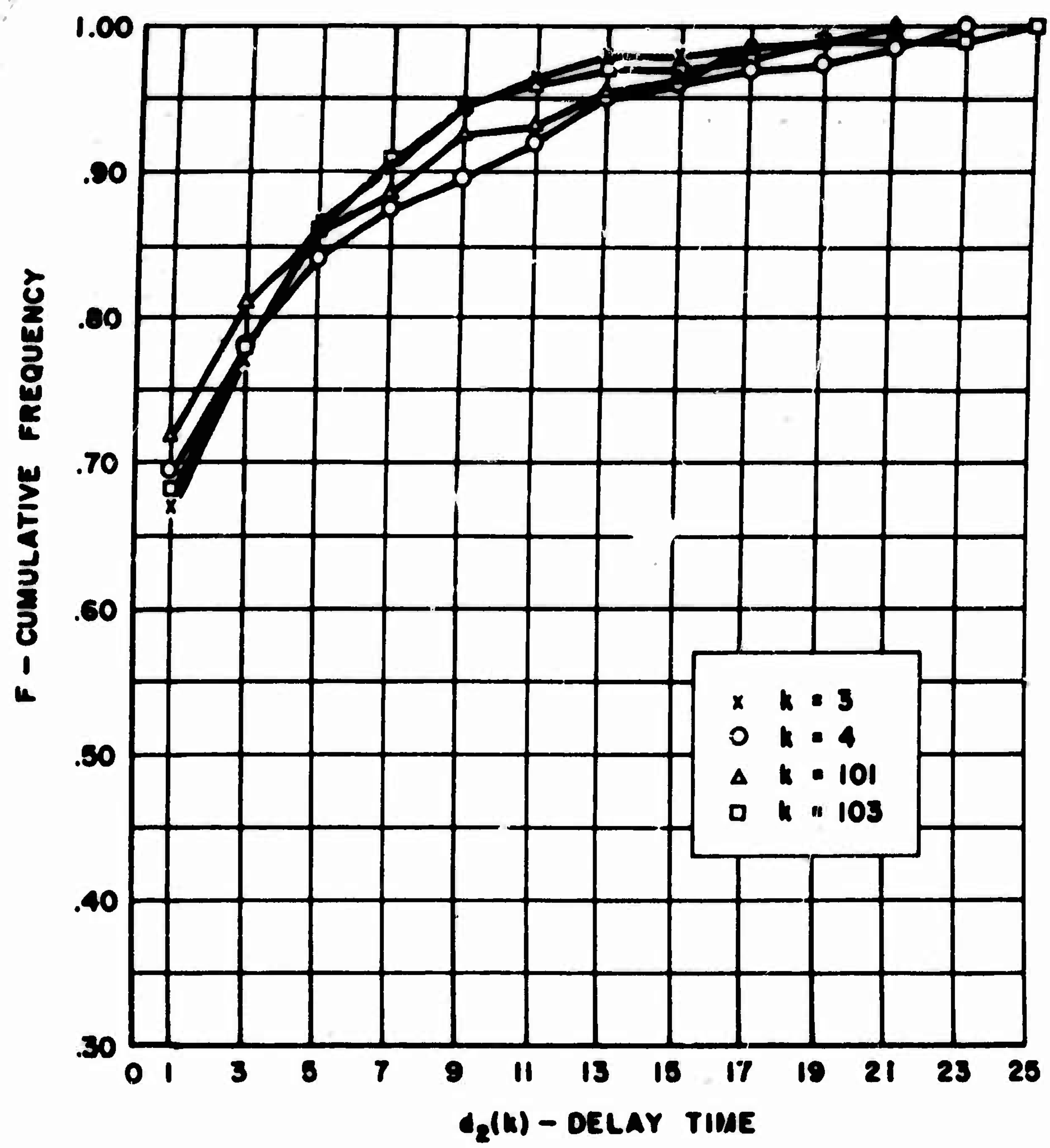


FIGURE 3 3-STAGE PROCESS DELAY - $d_2(k)$

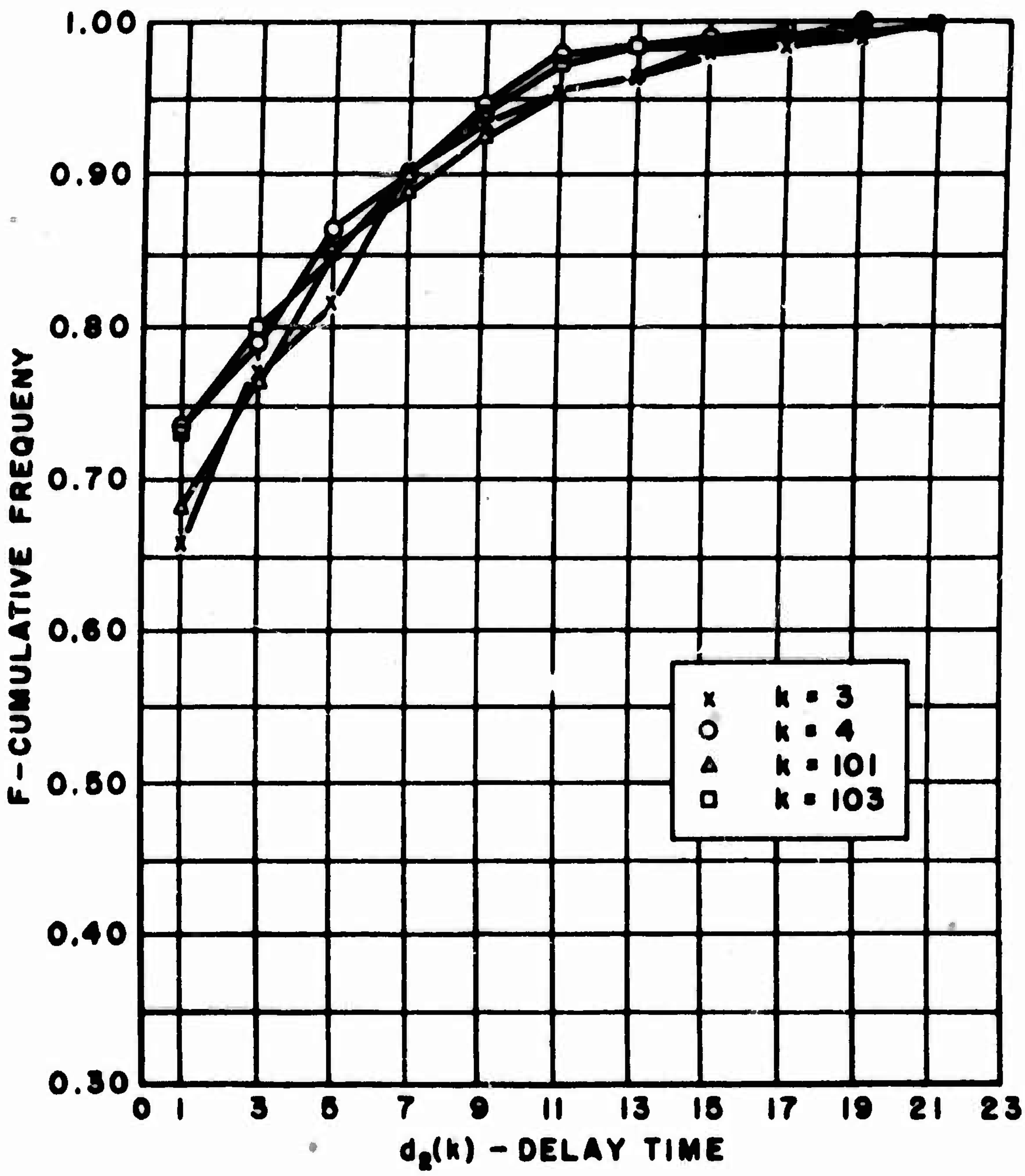


FIGURE 4 4-STAGE PROCESS DELAY - $d_2(k)$

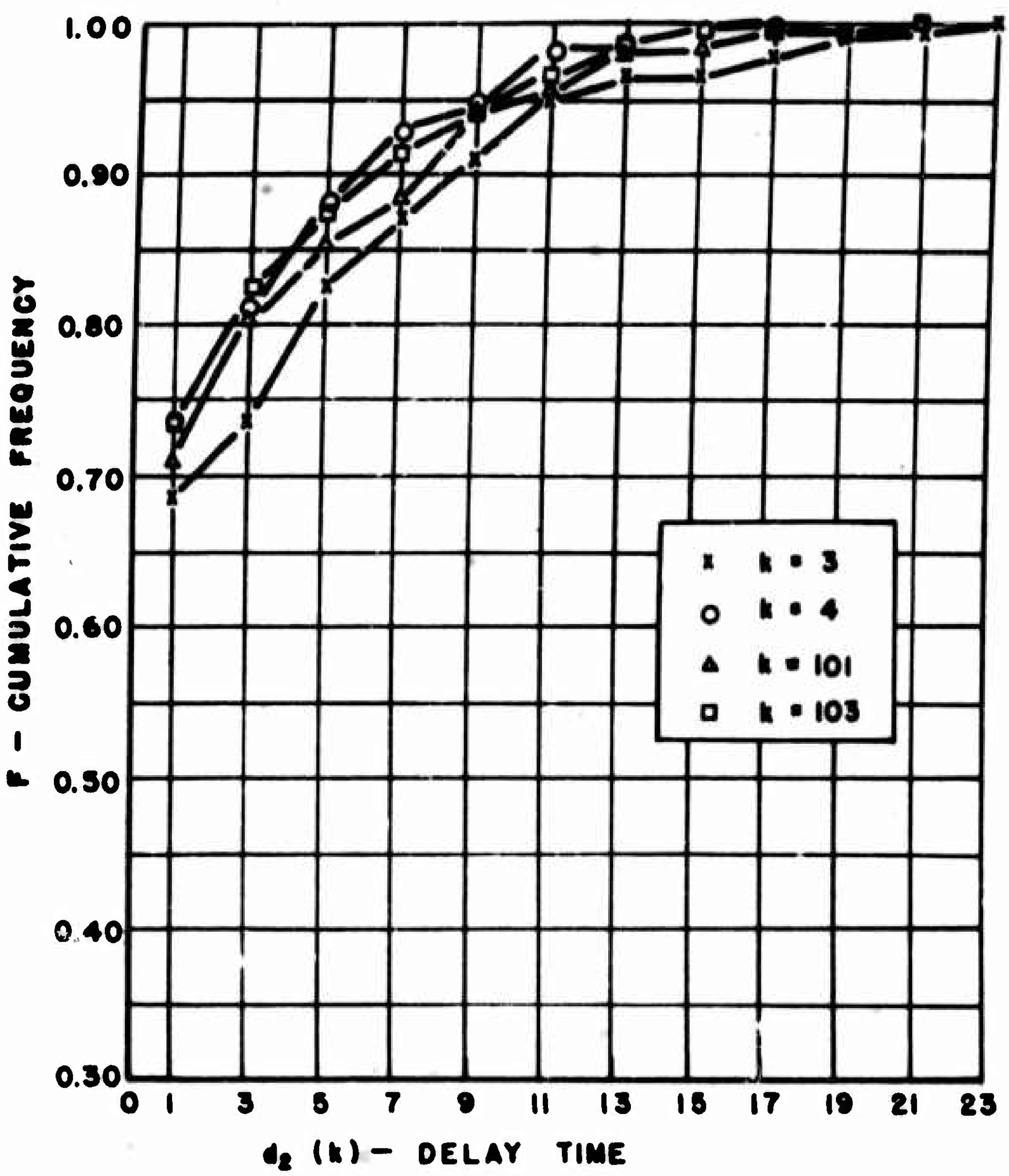


FIGURE 5 5 - STAGE PROCESS DELAY - $d_2(k)$

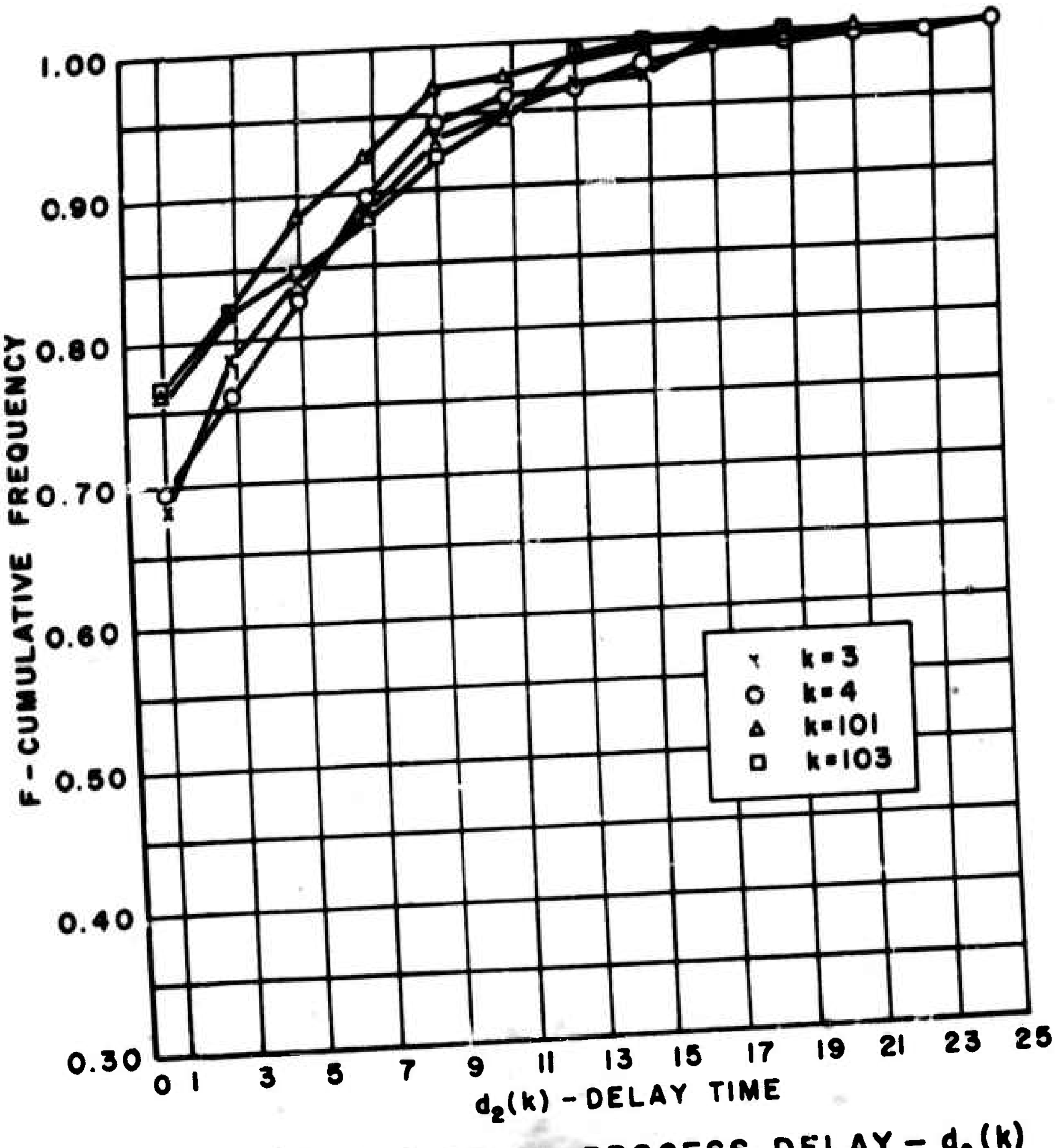


FIGURE 6 6-STAGE PROCESS DELAY - $d_2(k)$

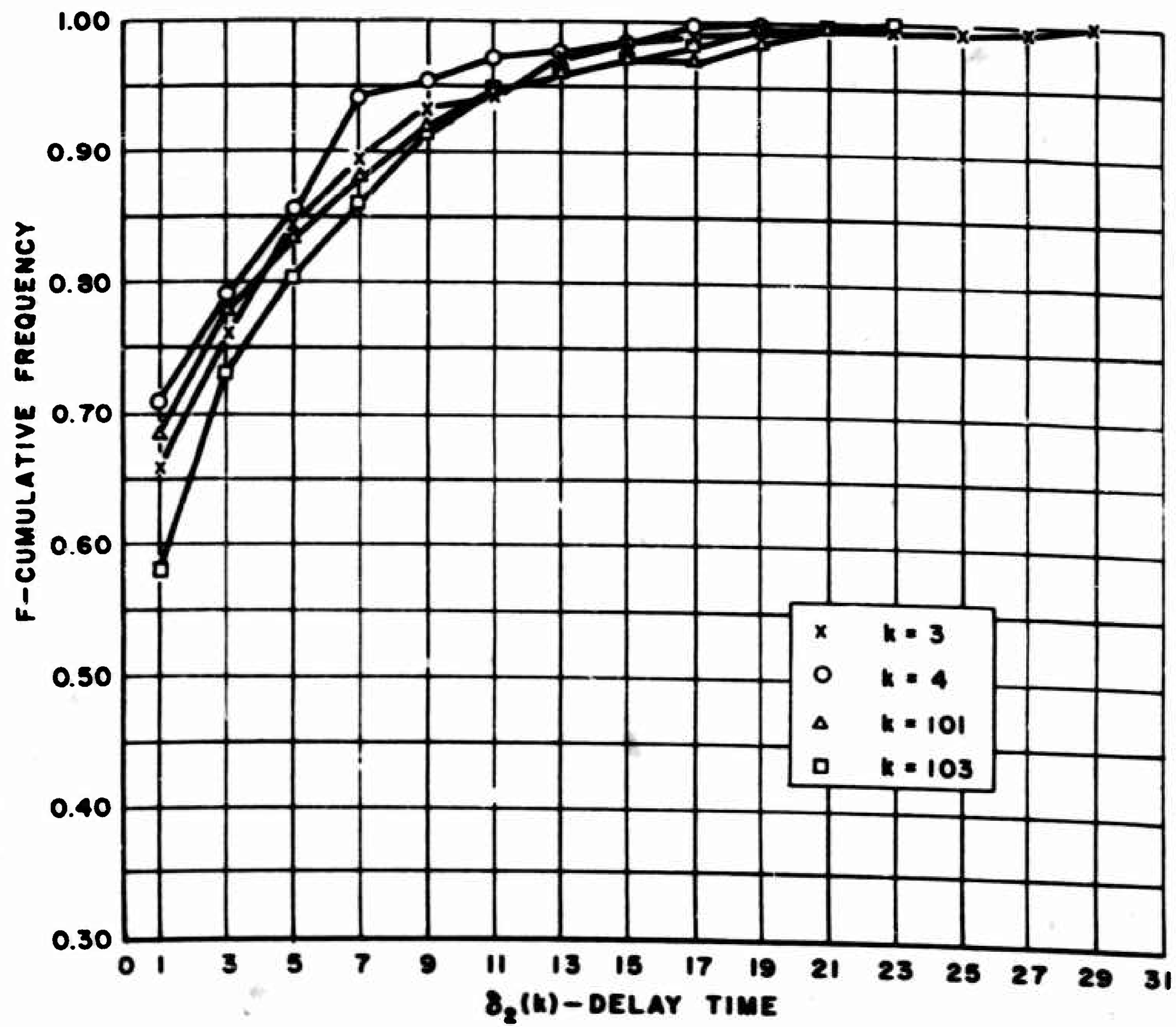


FIGURE 7 3-STAGE PROCESS DELAY – $\delta_2(k)$

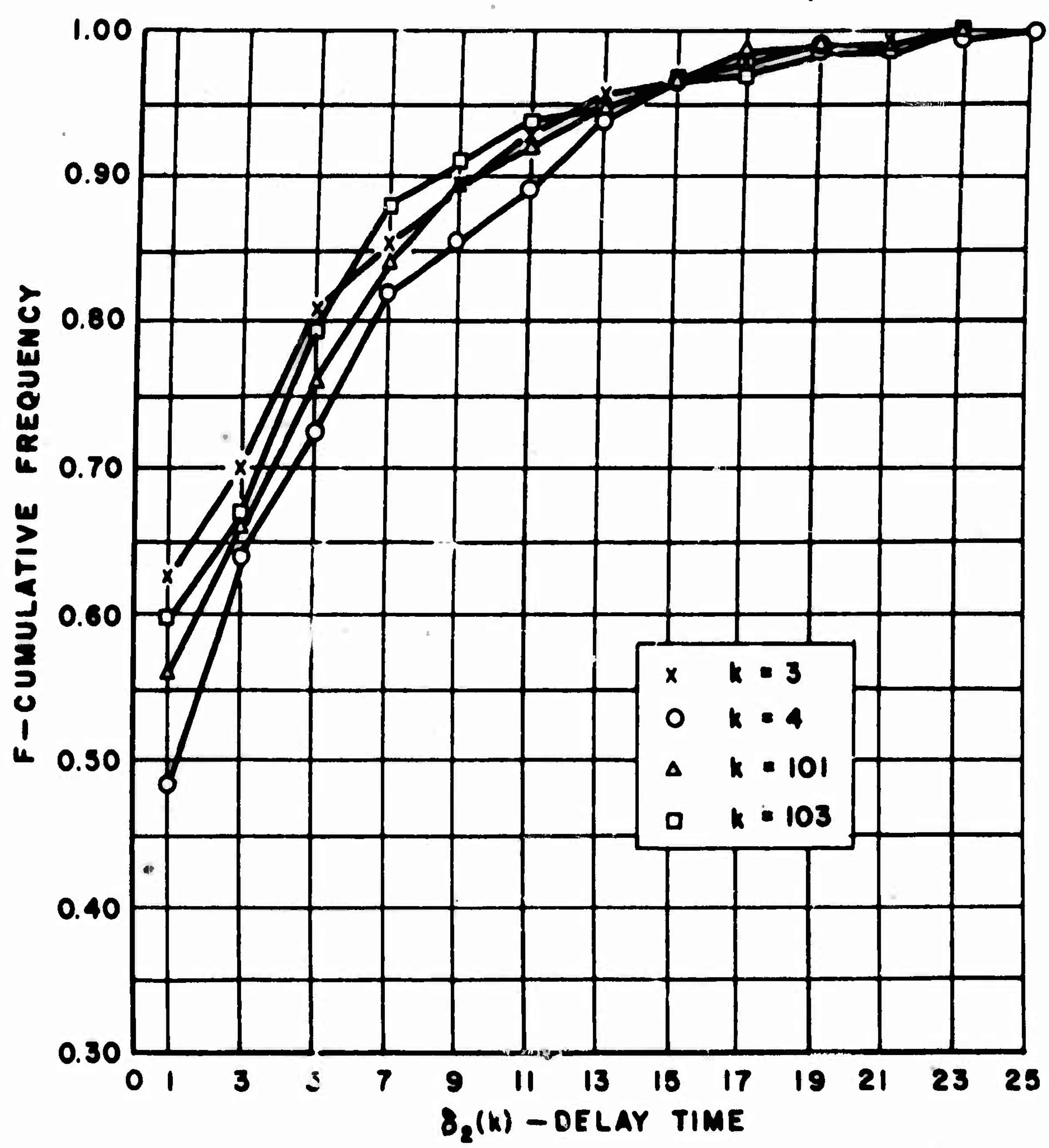


FIGURE 8 4-STAGE PROCESS DELAY – $\delta_2(k)$

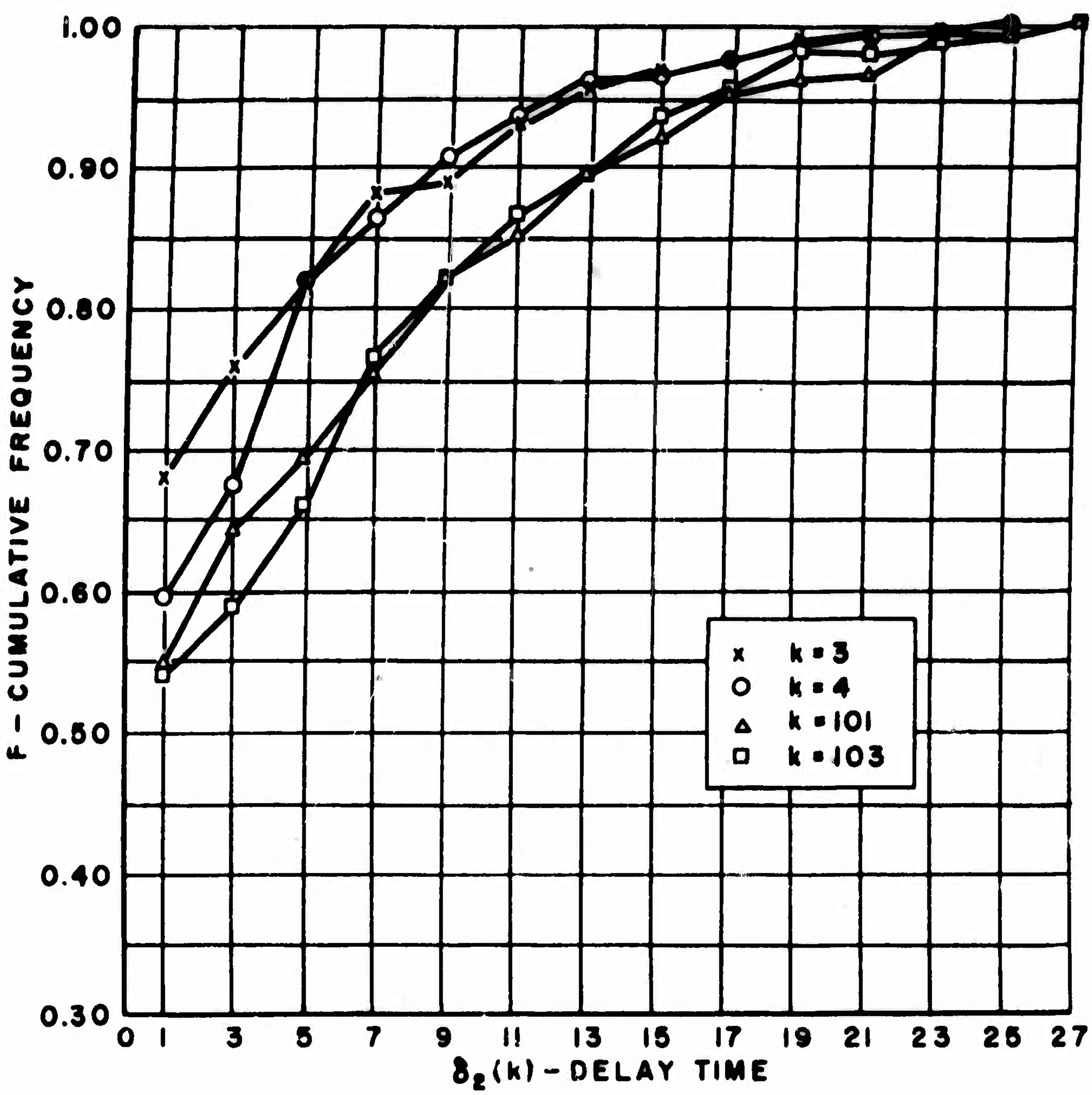


FIGURE 9 5-STAGE PROCESS DELAY - $\delta_2(k)$

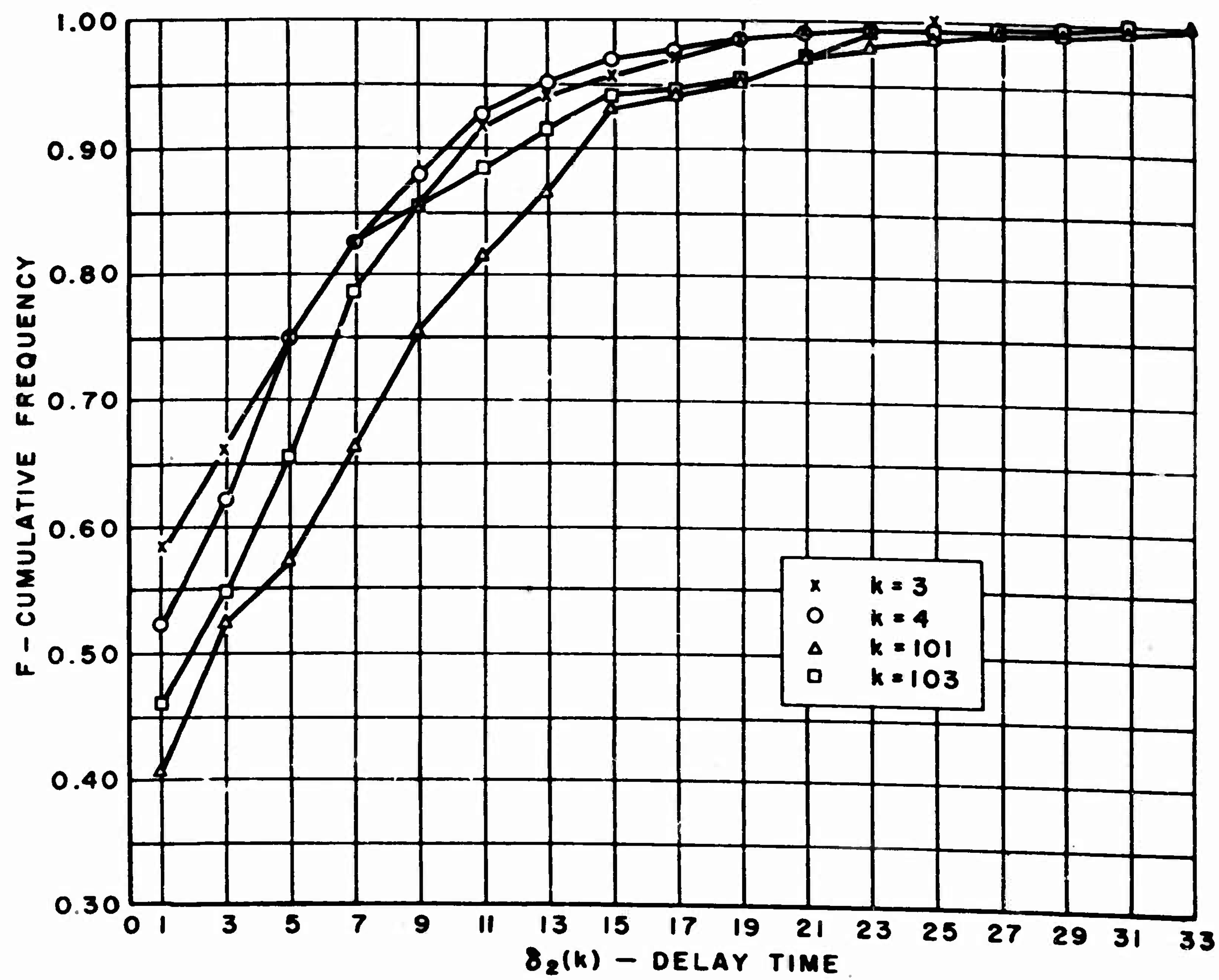


FIGURE 10 6-STAGE PROCESS DELAY - $\delta_2(k)$

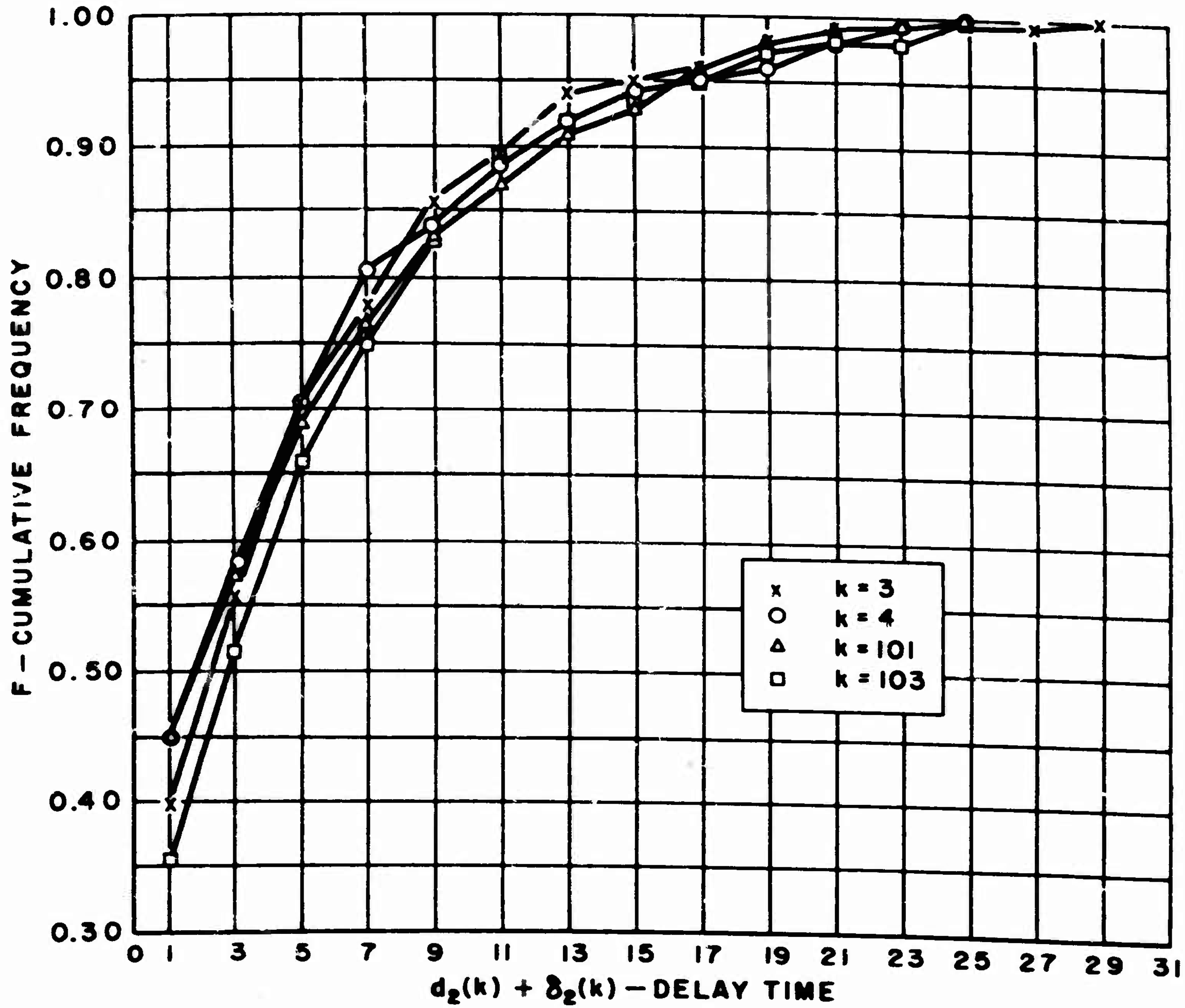


FIGURE II 3-STAGE PROCESS DELAY – $d_2(k) + \delta_2(k)$

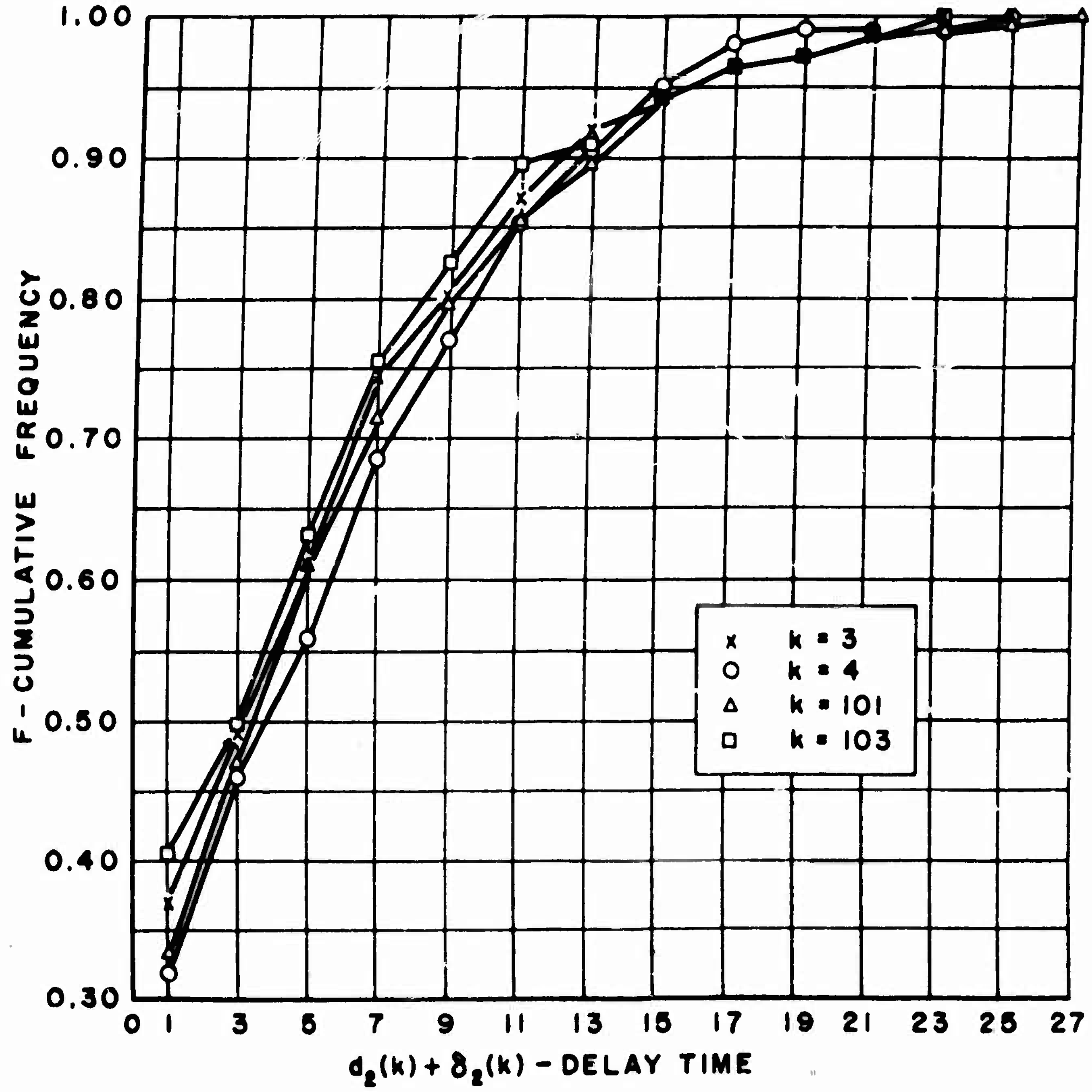


FIGURE 12 4-STAGE PROCESS DELAY — $d_2(k) + \delta_2(k)$

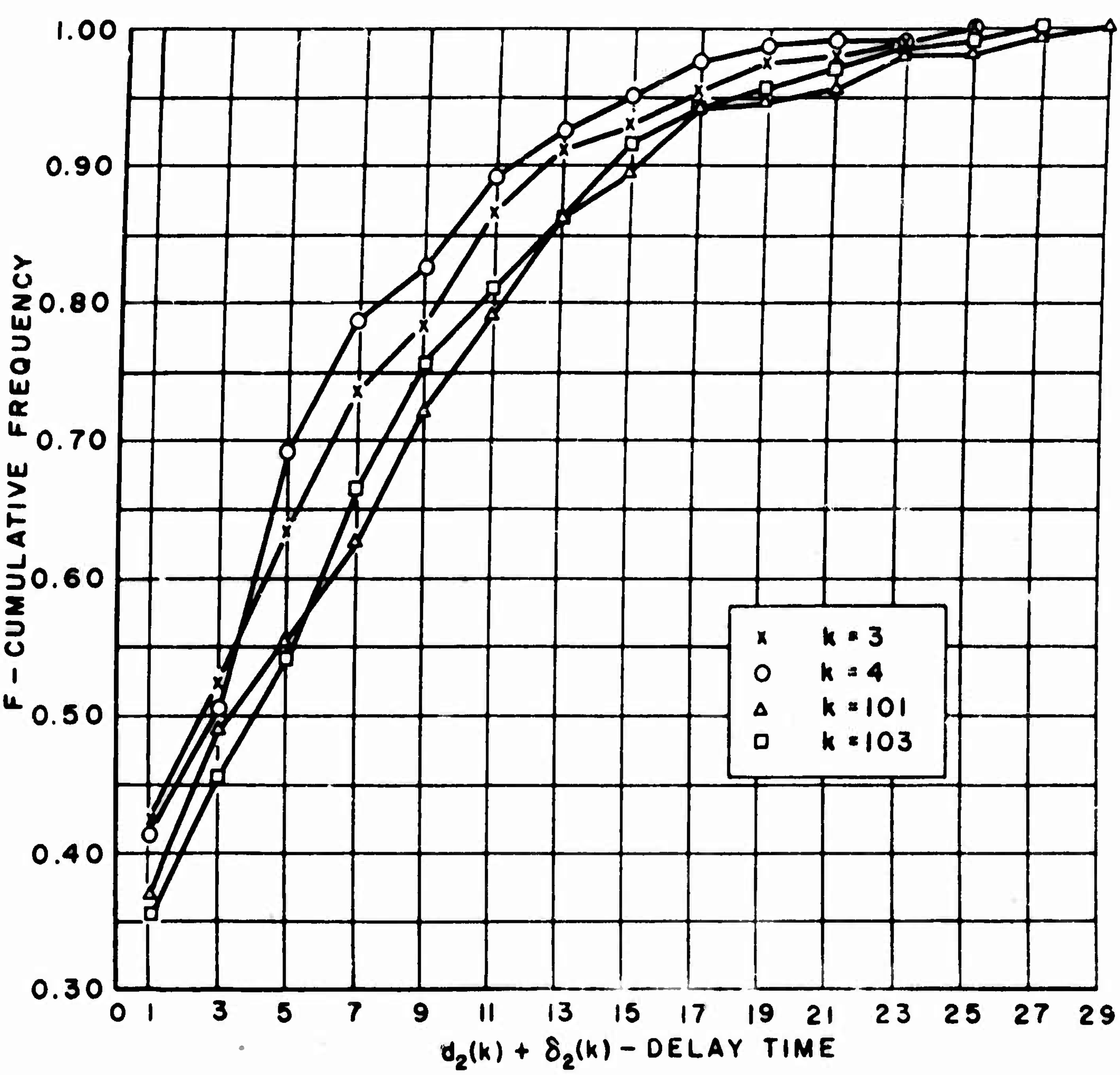


FIGURE 13 5-STAGE PROCESS DELAY – $d_2(k) + \delta_2(k)$

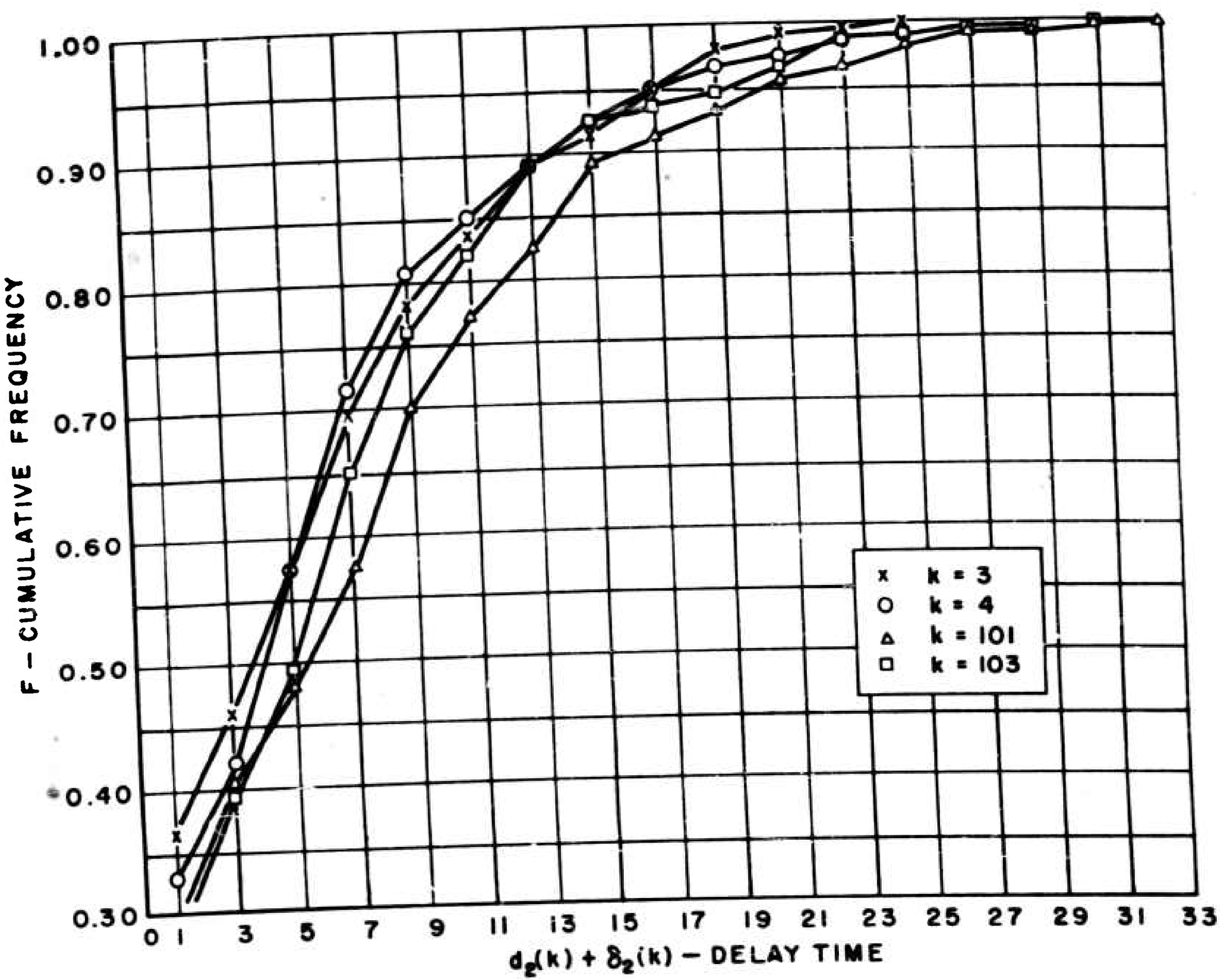


FIGURE 14 6-STAGE PROCESS DELAY – $d_2(k) + \delta_2(k)$

III - OUTLINE OF CODE

1. Simplified Flow Diagram For The 3-Stage Shuttle Process

Sub-Routine	Operation or Successor Criterion	Successor Yes	Successor No
A	Generate and store $t_1(k+1)$, $t_1'(k)$, $t_2(k)$, $t_2(k+1)$, $t_2'(k)$, $t_3(k)$, $t_3'(k)$.	B	
B	Prepare the answers $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$.	C	
C	Set tally for k ; is $k = 1$?	E	D
D	Replace the present $t_2(k)$ by the previous $t_2(k+1)$.	E	
E	Calculate $\alpha_k = t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k)$ and $d_2(k+1)$.	F	
F	Calculate $\beta_k = t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k)$ and $\delta_2(k+1)$.	G	
G	Set aside the present $t_2(k+1)$ for the following $t_2(k)$.	H	
H	Is $k \leq s$ (number of skips)?	A	I
I	Convert the prepared answers into the decimal system.	J	
J	Set tally for m ; is $m = 4$?	K	A
K	Punch out the answers; reset everything for the next repetition; set tally for r; is $r = 100$?	L	A
L	Halt.		

2. Simplified Flow Diagram For The 4-Stage Shuttle Process

Sub-Routine	Operation or Successor Criterion	Successor Yes	Successor No
A	Generate $t_1(k+1)$, $t_1'(k)$, $t_2(k)$, $t_2(k+1)$, $t_2'(k)$, $t_3(k)$, $t_3(k+1)$, $t_3'(k)$, $t_4(k)$, $t_4'(k)$.		B
B	Prepare the answers $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$.		C
C	Set tally for k; is $k = 1$?	E	D
D	Replace the present $t_2(k)$ and $t_3(k)$ by the previous $t_2(k+1)$ and $t_3(k+1)$.		E
E	Calculate $\alpha_k = t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k)$ and $d_2(k+1)$.		F
F	Calculate $\beta_k = t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k)$, $\delta_2(k+1)$ and $d_3(k+1)$.		G
G	Calculate $\gamma_k = t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k)$ and $\delta_3(k+1)$.		H
H	Set aside the present $t_2(k+1)$ and $t_3(k+1)$ for the following $t_2(k)$ and $t_3(k)$.		I
I	Is $k \leq s$ (number of skips)?	A	J
J	Convert the prepared answers into the decimal system.		K
K	Set tally for m; is $m = 4$?	L	A
L	Punch out the answers; reset everything for the next repetition; set tally for r; is $r = 100$?	M	A
M	Halt.		

3. Simplified Flow Diagram For The 5-Stage Shuttle Process

Sub-Routine	Operation or Successor Criterion	Successor Yes	Successor No
A	Generate $t_1(k+1)$, $t_1'(k)$, $t_2(k)$, $t_2(k+1)$, $t_2'(k)$, $t_3(k)$, $t_3(k+1)$, $t_3'(k)$, $t_4(k)$, $t_4(k+1)$, $t_4'(k)$, $t_5(k)$, $t_5'(k)$.	B	
B	Prepare the answers $d_2(k)$, $\delta_2(k)$, and $d_2(k) + \delta_2(k)$.	C	
C	Set tally for k; is $k = 1$?	E	D
D	Replace the present $t_2(k)$, $t_3(k)$, and $t_4(k)$ by the previous $t_2(k+1)$, $t_3(k+1)$ and $t_4(k+1)$.	E	
E	Calculate $\beta_1(k) = t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k)$ and $d_2(k+1)$.	F	
F	Calculate $\beta_2(k) = t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k)$, $\delta_2(k+1)$ and $d_3(k+1)$.	G	
G	Calculate $\beta_3(k) = t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k)$, $\delta_3(k+1)$ and $d_4(k+1)$.	H	
H	Calculate $\beta_4(k) = t_5(k) + t_5'(k) - t_4(k+1) - t_4'(k)$ and $\delta_4(k+1)$.	I	
I	Set aside the present $t_2(k+1)$, $t_3(k+1)$ and $t_4(k+1)$ for the following $t_2(k)$, $t_3(k)$ and $t_4(k)$.	J	
J	Is $k \leq s$ (number of skips)?	A	K
K	Convert the prepared answers into the decimal system.	L	
L	Set tally for m; is $m = 4$?	M	A

Sub-Routine	Operation or Successor Criterion	Successor Yes	Successor No
M	Punch out the answers; reset everything for the nex* repetition; set tally for r; is r = 100?	N	A
N	Halt.		

4. Simplified Flow Diagram For The 6-Stage Shuttle Process

Sub-Routine	Operation or Successor Criterion	Successor Yes	Successor No
A	Generate $t_1(k+1)$, $t_1'(k)$, $t_2(k)$, $t_2(k+1)$, $t_2'(k)$, $t_3(k)$, $t_3(k+1)$, $t_3'(k)$, $t_4(k)$, $t_4(k+1)$, $t_4'(k)$, $t_5(k)$, $t_5(k+1)$, $t_5'(k)$, $t_6(k)$, $t_6'(k)$.	B	
B	Prepare the answers $d_2(k)$, $\delta_2(k)$, $d_2(k) + \delta_2(k)$.	C	
C	Set tally for k; is k = 1?	E	D
D	Replace the present $t_2(k)$, $t_3(k)$, $t_4(k)$, $t_5(k)$ by the previous $t_2(k+1)$, $t_3(k+1)$, $t_4(k+1)$, $t_5(k+1)$.	E	
E	Calculate $\beta_1(k) = t_1(k+1) + t_1'(k) - t_2(k) - t_2'(k)$ and $d_2(k+1)$.	F	
F	Calculate $\beta_2(k) = t_3(k) + t_3'(k) - t_2(k+1) - t_2'(k)$, $\delta_2(k+1)$ and $d_3(k+1)$.	G	
G	Calculate $\beta_3(k) = t_4(k) + t_4'(k) - t_3(k+1) - t_3'(k)$, $\delta_3(k+1)$ and $d_4(k+1)$.	H	
H	Calculate $\beta_4(k) = t_5(k) + t_5'(k) - t_4(k+1) - t_4'(k)$, $\delta_4(k+1)$ and $d_5(k+1)$.	I	
I	Calculate $\beta_5(k) = t_6(k) + t_6'(k) - t_5(k+1) - t_5'(k)$ and $\delta_5(k+1)$.	J	
J	Set aside the present $t_2(k+1)$, $t_3(k+1)$, $t_4(k+1)$ and	K	

	$t_5(k+1)$ for the following $t_2(k)$, $t_3(k)$, $t_4(k)$ and $t_5(k)$.		
K	Is $k \leq s$ (number of skips)?	A	L
L	Convert the prepared answers into the decimal system.	M	
M	Set tally for m; is $m = 4$?	N	A
N	Punch out the answers; reset everything for the next repetition; set tally for r; is $r = 100$?	O	A
O	Halt.		

BIBLIOGRAPHY

1. R. Bellman, Technical Studies in Cargo Handling - 1, Formulation of Recurrence Equations for Shuttle Process and Assembly Line, Report 56-53, November 1956, Department of Engineering, University of California, Los Angeles.

2. R. R. O'Neill, An Engineering Analysis o Cargo Handling - V, Simulation of Cargo Handling Systems, Report 56-57, September 1956, Department of Engineering, University of California, Los Angeles.

APPENDIX A - CODE FOR 3 STAGE SHUTTLE PROCESS

000 - 009 SEE TABLE A-2

010	253	209	244	253	12
011	253	208	200	000	04
012	011	211	011	000	04
013	212	211	212	000	04
014	218	212	253	001	08
015	000	211	212	000	04
016	011	216	011	000	06
017	207	000	190	000	04
018	210	000	191	000	04
019	207	210	192	116	05
020	200	201	253	000	04
021	204	205	244	000	04
022	253	244	244	000	06
023	244	210	253	025	08
024	000	000	207	026	05
025	253	000	207	000	04
026	202	203	253	000	04
027	205	206	244	000	04
028	253	244	244	000	06
029	244	207	253	031	08
030	000	000	210	032	05
031	253	000	210	119	05
032	190	000	253	000	04
033	082	000	244	000	04
034	253	244	253	036	08
035	253	244	253	037	05
036	071	211	071	034	05
037	071	211	190	063	08
038	219	211	190	041	08
039	215	076	190	041	08
040	215	214	215	047	05
041	219	211	219	000	04
042	086	071	193	000	06
043	000	071	071	016	14
044	193	071	045	000	06
045	000	000	000	000	00
046	215	214	215	000	04
047	000	000	071	000	04
048	000	074	074	[illegible]	14
049	033	214	033	000	04
050	076	215	190	033	08
051	214	000	215	000	04
052	033	076	033	000	06
053	000	000	219	000	04
054	032	214	032	000	04
055	070	214	070	000	04
056	076	070	190	032	08
057	214	000	070	000	04
058	032	076	032	000	06
059	213	211	213	000	04
060	075	213	253	001	08
061	211	000	213	000	04
062	077	000	074	065	05
063	071	072	190	115	08
064	000	000	000	041	05
065	090	087	000	080	02
066	065	214	065	000	04
067	215	214	215	000	04
068	073	215	190	065	08
069	214	000	215	[illegible]	05
070	001	000	000	000	00
071	000	000	000	000	00
072	000	000	010	000	00
073	010	000	000	000	00
074	128	000	000	000	00
075	000	000	004	000	00
076	003	000	000	000	00
077	128	000	000	000	00
078	000	000	001	000	00
079	000	000	000	000	00
080	000	000	000	000	00
081	000	000	000	000	00
082	000	000	000	008	04
083	000	000	000	000	10
084	000	000	000	000	01
085	211	000	078	112	05
086	099	074	099	000	04
087	001	000	000	000	00
088	000	000	001	000	00
089	000	000	000	000	00
100	065	073	065	000	06
101	000	000	090	000	04
102	101	211	101	000	04
103	213	211	213	000	04
104	072	213	253	101	[illegible]
105	000	211	213	000	04
106	101	072	101	000	06
107	087	214	087	000	04
108	088	211	088	000	04
109	217	088	253	001	08
110	000	000	000	000	04
111	000	211	088	085	05
112	000	214	087	000	04
113	000	000	207	000	04
114	000	000	210	122	05
115	000	087	000	096	02
116	211	078	253	118	08
117	194	000	204	000	04
118	078	211	078	020	05
119	206	000	194	000	04
120	129	078	253	001	08
121	000	000	000	032	05
122	128	211	128	000	04

123	127	128	253	001	08
124	000	000	000	096	02
125	211	000	128	001	05
126	000	000	000	000	00
127	000	000	100	000	00
128	000	000	001	000	00

129 SEE TABLE A-1

200 * $x_1'(k)$

201 $x_1(k+1)$

202 $x_3(k)$

203 $x_3'(k)$

204 $x_2(k)$

205 $x_2'(k)$

206 $x_2(k+1)$

207 $\phi_2(k)$

208	000	000	000	000	00
209	000	000	000	000	01

210 $\delta_2(k)$

211	000	000	001	000	00
212	000	000	001	000	00
213	000	000	001	000	00
214	001	000	000	000	00
215	001	000	000	000	00
216	050	000	000	000	00
217	000	000	050	000	00
218	000	000	007	000	00
219	000	000	000	008	00

220 - 255 SEE TABLE A-3

TABLE A-1
CONSTANTS FOR SKIPS δ

129	000	000	000	000	00	$\delta = 0$
129	080	000	101	000	00	$\delta = 100$

TABLE A-2
GENERATION OF RANDOM VARIABLES

000	000	000	000	000	00
001	255	254	253	254	12
002	220	254	253	064	14
003	252	253	253	001	08
004	253	251	253	000	04
005	245	254	244	004	14
006	244	243	244	000	04
007	253	226	006	000	04
008	000	000	000	000	00
009	000	000	000	000	00

TABLE A-3
DENSITY FUNCTION AND CONSTANTS

220	248	255	255	255	15
221	000	255	255	255	15
222	255	000	255	255	15
223	255	255	000	255	15
224	255	255	255	000	15
225	255	255	255	255	00
226	000	244	009	000	04
227	007	003	003	001	01
228	007	003	003	001	01
229	007	003	003	001	01
230	007	005	003	001	01
231	007	005	003	001	01
232	007	005	003	001	01
233	009	005	003	001	01
234	009	005	003	001	01
235	009	005	003	001	01
236	009	005	003	001	01
237	011	005	003	001	01
238	011	005	003	001	01
239	011	005	003	001	01
240	013	005	003	001	01
241	013	005	003	001	01
242	015	007	003	017	01
243	000	227	000	000	00
244	000	000	000	000	00
245	255	240	255	255	15
246	225	000	253	064	14
247	224	000	253	088	14
248	223	000	253	076	14
249	222	000	253	084	14
250	221	000	253	092	14
251	251	000	000	000	00
252	002	000	000	000	00
253	000	000	000	000	00
254	004	140	039	057	05
255	004	140	039	057	05

* x AND x' CORRESPOND TO ξ AND ξ' IN REPORT.

APPENDIX B - CODE FOR 4 STAGE SHUTTLE PROCESS

000-009 SEE TABLE A-2

010	253	219	244	253	12
011	253	218	090	000	04
012	011	217	011	000	04
013	216	217	216	000	04
014	215	216	244	001	08
015	217	000	216	000	04
016	011	215	011	000	06
017	110	000	114	000	04
018	111	000	115	000	04
019	110	111	116	000	04
020	217	214	244	023	08
021	117	000	092	000	04
022	116	000	095	000	04
023	090	091	253	000	04
024	092	093	244	000	04
025	253	244	253	000	06
026	253	111	110	028	08
027	000	000	110	000	04
028	093	094	253	000	04
029	095	096	246	000	04
030	253	244	253	000	06
031	253	110	253	000	04
032	112	253	111	035	08
033	000	111	113	000	06
034	000	000	111	036	05
035	000	000	113	000	04
036	096	097	253	000	04
037	098	099	244	000	04
038	244	253	253	000	06
039	253	113	112	041	08
040	000	000	112	000	04
041	094	000	117	000	04
042	097	000	118	213	05
043	120	214	244	001	08
044	114	000	244	000	04
045	210	000	253	000	04
046	244	253	244	048	08
047	244	253	244	049	05
048	121	217	121	046	05
049	121	215	119	208	08
050	205	121	119	000	06
051	000	121	121	016	14
052	119	121	053	000	06
053	000	000	000	000	00
054	204	203	204	000	04
055	045	203	045	000	04
056	000	206	206	065	14
057	000	000	121	000	04
058	202	204	119	045	08
059	203	000	204	000	04
060	045	202	045	000	06
061	044	203	044	000	04
062	122	203	122	000	04
063	252	122	119	044	08
064	000	000	122	000	04
065	044	202	044	000	06
066	123	203	123	000	04
067	202	123	119	001	08
068	000	000	123	000	04
069	100	124	000	080	02
070	069	203	069	000	04
071	204	203	204	000	04
072	201	204	119	069	08
073	203	000	204	000	04
074	069	201	069	000	06
075	000	000	100	000	04
076	075	217	075	000	04
077	216	217	216	000	04
078	215	216	119	075	08
079	217	000	216	000	04
080	075	215	075	000	06
081	207	000	206	000	04
082	124	203	124	000	04
083	130	124	119	001	08
084	000	000	124	000	04
085	217	000	214	000	04
086	000	000	110	000	04
087	000	000	111	000	04
088	000	000	112	000	04
089	000	000	113	190	05

090 * $x_1'(k)$
091 $x_1(k+1)$
092 $x_2(k)$
093 $x_2'(k)$
094 $x_2(k+1)$
095 $x_3(k)$
096 $x_3'(k)$
097 $x_3(k+1)$
098 $x_4(k)$
099 $x_4'(k)$
110 $d_2(k)$
111 $\delta_2(k)$
112 $d_3(k)$
113 $\delta_3(k)$
120 SEE TABLE B-1

130	000	000	000	000	00
140	099	000	000	000	00
190	125	203	125	000	04
191	140	125	119	001	08
192	000	000	000	096	02
193	000	000	125	001	05

* x AND x' CORRESPOND TO f AND f' IN REPORT

201	010	000	000	000	00
202	003	000	000	000	00
203	001	000	000	000	00
204	001	000	000	000	00
205	109	6	109	000	04
206	128	000	000	000	00
207	128	000	000	000	00
208	244	253	244	000	04
209	100	206	100	054	05
210	000	000	000	006	04
211	000	000	000	000	10
212	000	000	000	000	01
213	214	217	214	043	05
214	000	000	001	000	00
215	000	000	010	000	00
216	000	000	001	000	00
217	000	000	001	000	00
218	010	000	000	000	00
219	000	000	000	000	01

220-255 SEE TABLE A-3

TABLE B-1
CONSTANTS FOR SKIPS S

120	000	000	001	000	00	$S = 0$
120	000	000	101	000	00	$S = 100$

APPENDIX C - CODE FOR 5 STAGE SHUTTLE PROCESS

000-009 SEE TABLE A-2

010	253	218	244	253	12
011	253	219	110	000	04
012	011	217	011	000	04
013	216	217	216	000	04
014	215	216	253	001	08
015	217	000	216	000	04
016	011	215	011	000	06
017	123	000	129	000	04
018	124	000	130	000	04
019	123	124	131	000	04
020	000	136	253	024	08
021	132	000	110	000	04
022	133	000	114	000	04
023	134	000	117	000	04
024	110	111	253	000	04
025	112	113	244	000	04
026	244	253	244	000	06
027	244	124	123	029	08
028	000	000	123	030	04
029	114	115	253	000	04
030	111	116	244	000	04
031	253	244	244	000	06
032	244	126	244	000	04
033	244	123	124	036	08
034	000	124	125	000	06
035	000	000	124	037	05
036	000	000	125	000	04
037	117	118	253	000	04
038	115	119	244	000	04
039	253	244	244	000	06
040	244	128	244	000	04
041	244	125	126	044	08
042	000	126	127	000	06
043	000	000	126	045	05
044	000	000	127	000	04
045	120	121	253	000	04
046	118	122	244	000	04
047	253	244	244	000	06
048	244	127	128	050	08
049	000	000	128	000	04
050	116	000	132	000	04
051	119	000	133	000	04
052	122	000	134	000	04
053	136	217	136	000	04
054	138	136	135	001	08
055	129	000	253	000	04
056	211	000	244	000	04
057	253	244	253	059	08
058	253	244	253	060	05
059	139	217	139	057	05
060	139	210	135	208	08
061	205	139	135	000	06
062	000	139	139	016	14
063	135	139	064	000	06
064	000	000	000	000	00
065	204	203	204	000	04
066	000	207	207	065	14
067	056	203	056	000	04
068	000	000	139	000	04
069	202	204	135	056	08
070	203	000	204	000	04
071	056	202	056	000	06
072	055	203	055	000	04
073	137	203	137	000	04
074	252	137	135	055	08
075	000	000	137	000	04
076	055	202	055	000	06
077	141	203	141	000	04
078	202	141	135	001	08
079	000	000	141	000	04

080	206	000	207	000	04
081	100	000	000	080	02
082	081	203	081	000	04
083	204	203	204	000	04
084	201	204	135	081	08
085	203	000	204	000	04
086	081	201	081	000	06
087	000	000	100	000	04
088	087	217	087	000	04
089	216	217	216	000	04
090	210	216	135	087	08
091	217	000	216	000	04
092	087	210	087	194	07
093	000	000	136	000	04
094	000	000	123	000	04
095	000	000	124	000	04
096	000	000	125	000	04
097	000	000	126	000	04
098	000	000	127	000	04
099	000	000	128	190	05

110 * $x_2(k)$
111 $x'_2(k)$
112 $x'_1(k)$
113 $x_1(k+1)$
114 $x_3(k)$
115 $x'_3(k)$
116 $x_2(k+1)$
117 $x_4(k)$
118 $x'_4(k)$
119 $x_3(k+1)$
120 $x_5(k)$
121 $x'_5(k)$
122 $x_4(k+1)$
123 $d_2(k)$
124 $\delta_2(k)$
125 $d_3(k)$
126 $\delta_3(k)$
127 $d_4(k)$
128 $\delta_4(k)$

138 SEE TABLE C-1

140	000	000	099	000	00
150	000	000	000	000	00
190	142	217	142	000	04
191	140	142	135	001	08
192	000	000	000	096	02
193	000	000	142	001	05
194	143	217	143	000	04
195	150	143	135	001	08
196	000	000	143	093	05

* x AND x' CORRESPOND TO f AND f' IN REPORT

201	010	000	000	000	00
202	003	000	000	000	00
203	001	000	000	001	00
204	001	000	000	000	00
205	109	207	109	000	04
206	128	000	000	000	00
207	128	000	000	000	00
208	100	207	100	000	04
209	253	244	253	065	05
210	000	000	010	000	00
211	000	000	000	008	04
212	000	000	000	000	10
213	000	000	000	000	01
214	000	000	001	000	00
215	000	000	013	000	00
216	000	000	001	000	00
217	000	000	001	000	00
218	000	000	000	000	01
219	000	000	000	000	00

220 - 255 SEE TABLE A-3

TABLE C-1
CONSTANTS FOR SKIPS S

S = 0	136	000	000	000	000	00
S = 100	138	000	000	100	000	00

APPENDIX D - CODE FOR 6 STAGE SHUTTLE PROCESS

000 - 009 SEE TABLE A-2

010	253	219	244	253	12
011	253	218	110	000	04
012	011	217	011	000	04
013	216	217	216	000	04
014	215	216	253	001	08
015	217	000	216	000	04
016	011	215	011	000	06
017	126	000	134	000	04
018	127	000	135	000	04
019	126	[illegible]	136	000	04
020	000	142	253	025	08
021	137	000	110	000	04
022	138	000	114	000	04
023	139	000	117	000	04
024	140	000	120	000	04
025	110	111	253	000	04

026	112	113	244	000	04
027	244	253	244	000	06
028	244	127	126	030	08
029	000	000	126	000	04
030	114	115	253	000	04
031	111	116	244	000	04
032	253	244	244	000	06
033	244	129	253	000	04
034	253	126	127	037	08
035	000	127	126	000	06
036	000	000	127	038	05
037	000	000	128	000	04
038	117	118	253	000	04
039	115	119	244	000	04
040	253	244	253	000	06
041	253	131	253	000	04
042	253	128	129	045	08
043	000	129	130	000	06
044	000	000	129	046	05
045	000	000	130	000	04
046	120	121	253	000	04
047	116	122	244	000	04
048	253	244	253	000	06
049	253	133	253	000	04
050	253	130	131	053	08
051	000	131	132	000	06
052	000	000	131	054	05
053	000	000	132	000	04
054	123	124	253	000	04
055	121	125	244	000	04
056	253	244	253	000	06
057	253	132	133	059	08
058	000	000	133	000	04
059	116	000	137	000	04
060	119	000	138	000	04
061	122	000	139	000	04
062	125	000	140	000	04
063	142	217	142	000	04
064	146	142	253	001	08
065	134	000	253	000	04
066	212	000	244	000	04
067	253	244	253	069	08
068	253	244	253	070	05
069	143	217	143	067	05
070	211	143	141	209	08
071	206	143	141	000	06
072	000	143	143	016	14
073	141	143	074	000	06
074	000	000	000	000	00
075	205	204	205	000	04
076	066	204	066	000	04
077	000	000	143	000	04
078	000	207	207	065	14
079	203	205	141	066	08
080	204	000	205	085	05
081	065	204	065	000	04
082	144	204	144	000	04
083	252	144	141	065	08
084	000	000	144	086	05
085	066	203	066	081	07
086	065	203	065	000	06
087	145	204	145	000	04
088	203	145	141	001	08
089	000	000	145	000	04
090	206	000	207	000	04
091	100	143	000	080	02
092	091	204	091	000	04
093	205	204	205	000	04
094	202	205	141	091	08
095	204	000	205	000	04
096	091	202	091	000	06
097	000	000	100	000	04
098	097	217	097	000	04
099	216	217	216	190	05

110 *$x_2(k)$
111 $x'_2(k)$
112 $x'_1(k)$
113 $x_1(k+1)$
114 $x_3(k)$
115 $x'_3(k)$
116 $x_2(k+1)$
117 $x_4(k)$
118 $x'_4(k)$
119 $x_3(k+1)$
120 $x_5(k)$
121 $x'_5(k)$
122 $x_4(k+1)$
123 $x_6(k)$
124 $x'_6(k)$
125 $x_5(k+1)$
126 $d_2(k)$
127 $\delta_2(k)$
128 $d_3(k)$
129 $\delta_3(k)$
130 $d_4(k)$
131 $\delta_4(k)$
132 $d_5(k)$
133 $\delta_5(k)$

148 SEE TABLE D-1

150	000	000	000	000	00
160	099	000	000	000	00
170	197	179	197	000	06
171	147	204	147	000	04

*X AND X' CORRESPOND TO † AND †' IN REPORT

172	160	147	141	001	04
173	000	000	000	004	02
174	003	000	147	001	03
179	000	000	000	000	00
180	211	216	141	097	00
181	217	000	216	000	04
182	247	211	047	000	00
183	146	204	140	000	04
184	130	140	141	001	00
185	005	200	140	000	04
186	000	000	142	000	04
187	003	000	136	000	04
188	147	217	147	000	04
189	216	217	216	000	04
200	174	216	141	147	00
201	217	000	216	170	05
202	010	000	000	000	00
203	001	000	000	000	00
204	001	000	000	000	00
205	001	046	000	000	00
206	126	000	000	000	00
207	124	000	003	000	00
208	100	207	100	000	04
209	213	444	203	000	04
210	100	407	100	173	05
211	000	000	010	000	00
212	000	000	000	004	04
213	000	000	000	000	10
214	000	000	000	000	01
215	000	000	016	000	00
216	000	000	001	000	00
217	000	000	001	000	00
218	000	000	000	000	00
219	000	000	000	000	01

220-255 SEE TABLE A-3

TABLE D-1
CONSTANTS FOR SKIPS S

140	000	000	000	000	00	S=0
140	000	000	100	000	00	S=100